d. $y = ax^b$ Write the untranslated general equation.

$\begin{cases} 44.8 = a \cdot 4^b \\ 151.2 = a \cdot 6^b \end{cases}$ Substitute the given x- and y-values into the equation.

$\frac{151.2}{44.8} = \frac{a \cdot 6^b}{a \cdot 4^b}$ Divide the second equation by the first to eliminate a.

$3.375 = 1.5^b$ The a's cancel, and $\frac{6^b}{4^b} = \left(\frac{6}{4}\right)^b = 1.5^b$.

$\log 3.375 = \log 1.5^b$ Take the logarithm of both sides to get b out of the exponent. See Section 2-4 for a review of logarithms.

$\log 3.375 = b \log 1.5$

$b = \frac{\log 3.375}{\log 1.5} = 3$

$44.8 = a \cdot 4^3$ Substitute 3 for b in one of the equations.

$a = \frac{44.8}{4^3} = 0.7$

$\therefore y = 0.7x^3$ Write the particular equation.

e. Plotting the graph confirms that the equation is correct. Note that the value of b is between 0 and 1, which corresponds to the fact that the graph is concave up. ◀

EXAMPLE 4 ➤ For the function graphed in Figure 2-2i,

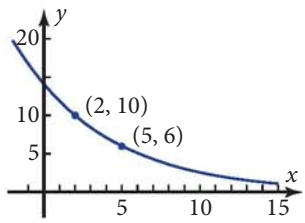

Figure 2-2i

a. Identify the kind of function it could be.

b. On what interval or intervals is the function increasing or decreasing? Which way is the graph concave, up or down?

c. Describe something in the real world that a function with this shape graph could model.

d. Find the particular equation of the function, given that the points (2, 10) and (5, 6) are on the graph.

e. Confirm by plotting that your equation gives the graph in Figure 2-2i.

SOLUTION

a. The function could be exponential or quadratic, but exponential is chosen because the graph appears to approach the x-axis asymptotically.

b. The function is decreasing and concave up over its entire domain.

c. The function could model any situation in which a variable quantity starts at some nonzero value and decreases, gradually approaching zero, such as the number of degrees a cup of coffee is above room temperature as a function of time since it started cooling.